HIGH SCHOOL AND UNIVERSITY MATRICULATION EXAMINATIONS BOARD.

DEPARTMENTAL EXAMINATIONS, 1934.

ALGEBRA 3.

(Crawford)

Time—Two and one-half hours.

N.B.—Candidates are directed to write on the answer-envelope the word "Crawford".

Values.

1. Attempt *three* parts only:

8 (*a*) The first term of an arithmetic progression is 2, and the common difference is 1. How many terms must be taken so that the sum may be 65?

8 (*b*) Find the sum of n terms, and the sum to infinity of the series

$$1 - x + x^2 - x^3 + x^4 - \ldots\ldots (0 < x < 1).$$

8 (*c*) A geometric progression has 20 terms, of which the second is 25 and the fourth 16. Find the sum of the progression.

8 (*d*) If $\frac{x}{x-y} + \frac{z}{z-y} = 0$, prove that x, y and z are in harmonic progression.

2. Attempt *two* parts only:

8 (*a*) Find the sum of n terms, and the sum to infinity, of the following series:

$$1 + \tfrac{3}{4} + \tfrac{5}{16} + \tfrac{7}{64} + \ldots\ldots\ldots\ldots$$

8 (*b*) If $4x^2 - bx + 3 = 0$, find the value of b such that one root of the equation will be 3 times the other.

8 (*c*) If $\frac{a}{x+y-z} = \frac{b}{y+z-x} = \frac{c}{z+x-y}$, show that $\frac{x}{c+a} = \frac{y}{a+b} = \frac{z}{b+c}$.

Values.

10 **3.** The force required to punch a hole through an iron plate varies as the diameter of the hole for a plate of given thickness, and varies as the thickness for a hole of given diameter. What is the ratio of the force required to punch a hole of $\frac{2}{3}$ in. diameter through a $\frac{1}{4}$ in. plate to the force needed to punch a hole of $\frac{5}{8}$ in. diameter through a $\frac{9}{16}$ in. plate?

4. Attempt *two* parts only:

8 (*a*) Twelve points are taken on the circumference of a circle. How many triangles are there which have three of these points as vertices?

8 (*b*) A bag contains 8 red and 12 white balls. In how many ways may 3 red and 2 white balls be drawn from the bag?

8 (*c*) Obtain the formula for the number of combinations of n different things taken r at a time.

5. Attempt *three* parts only:

8 (*a*) Write the first five terms of the binomial series for $\dfrac{1}{(1+x)^3}$.

8 (*b*) Find the term containing x^{32} in the expansion of $\sqrt{1-x^2}$ by the binomial series.

8 (*c*) What is the coefficient of x^2 in the expansion of $(1-x+x^2)(1-x)^7$?

8 (*d*) Prove that

$$1+\frac{1}{6}+\frac{1.3}{1.2}\cdot\frac{1}{6^2}+\frac{1.3.5}{1.2.3}\cdot\frac{1}{6^3}+\ldots\ldots=\sqrt{\frac{3}{2}}.$$

10 **6.** An employee will have to retire at 65. Each year beginning when he is 50 years of age, he puts aside $500, which he invests at 5% compound interest. What will be the amount of his savings when he retires?

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